TRUNG TÂM HOCMAI.ONLINE
P.2512 – 34T – Hoàng Đạo Thúy Tel: (094)-2222-408

Hà Nội, ngày 08 tháng 05 năm 2010

BTVN NGÀY 08-05

Giải các phương trình lượng giác sau:

1/ $2\cos 2x - 8\cos x + 7 = \dfrac{1}{\cos x}$

2/ $4\cos^2 x + 3\tan^2 x - 4\sqrt{3}\cos x + 2\sqrt{3}\tan x + 4 = 0$

3/ $\sqrt{3-\cos x} - \sqrt{\cos x + 1} = 2$

4/ $S\text{in}^3 x - \cos^3 x = \cos 2x.\tan\left(x+\dfrac{\pi}{4}\right).\tan\left(x-\dfrac{\pi}{4}\right)$

5/ $Cos^2\left(x+\dfrac{\pi}{3}\right) + Cos^2\left(x+\dfrac{2\pi}{3}\right) = \dfrac{1}{2}(\sin x + 1)$

.....................**Hết**.....................

Phụ trách môn Toán hocmai.vn

Trịnh Hào Quang

HDG CÁC BTVN

- **BTVN NGÀY 05-05:**

1/ $4\sin^3 x - 1 = 3\sin x - \sqrt{3}\cos 3x$

$$\Leftrightarrow \sin 3x - \sqrt{3}\cos 3x = -1 \Leftrightarrow \frac{1}{2}\sin 3x - \frac{\sqrt{3}}{2}\cos 3x = -\frac{1}{2}$$

$$\Leftrightarrow \sin\left(3x - \frac{\pi}{3}\right) = \sin\left(-\frac{\pi}{6}\right) \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{18} + \frac{k2\pi}{3} \\ x = \frac{\pi}{2} + \frac{k2\pi}{3} \end{array}\right.$$

2/ $\sin 3x + (\sqrt{3} - 2)\cos 3x = 1$

$$Coi: t = \tan\frac{3x}{2} \Rightarrow \frac{2t}{1+t^2} + \frac{(\sqrt{3}-2)(1-t^2)}{1+t^2} = 1 \Leftrightarrow (\sqrt{3}-1)t^2 - 2t + (3-\sqrt{3}) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} t = 1 \\ t = \sqrt{3} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \tan\frac{3x}{2} = 1 \\ \tan\frac{3x}{2} = \sqrt{3} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{6} + \frac{k2\pi}{3} \\ x = \frac{2\pi}{9} + \frac{k2\pi}{3} \end{array}\right.$$

3/ $4\sin^3 x + 3\cos^3 x - 3\sin x - \sin^2 x\cos x = 0 (1)$

$*$Xét $\sin x = 0 \Rightarrow 3\cos^3 x = \pm 3 \neq 0$

$(1) \Leftrightarrow 4 + 3\cot^3 x - 3(\cot^2 x + 1) - \cot x = 0$

$$\Leftrightarrow \left[\begin{array}{l} \cot x = 1 \\ \cot x = -\frac{1}{\sqrt{3}} \\ \cot x = \frac{1}{\sqrt{3}} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{4} + k\pi \\ x = \pm\frac{\pi}{3} + k\pi \end{array}\right.$$

4/ $2\sin 5x+\sqrt{3}\cos 3x+\sin 3x=0$

$$\sqrt{3}\cos 3x+\sin 3x=-2\sin 5x \Leftrightarrow -\frac{\sqrt{3}}{2}\cos 3x-\frac{1}{2}\sin 3x=\sin 5x$$

$$\Leftrightarrow \cos\left(\frac{5\pi}{6}+3x\right)=\sin 5x=\cos\left(\frac{\pi}{2}-5x\right)$$

$$\Leftrightarrow \left[\begin{array}{l}\frac{5\pi}{6}+3x=\frac{\pi}{2}-5x+k2\pi\\ \frac{5\pi}{6}+3x=5x-\frac{\pi}{2}+k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=-\frac{\pi}{24}+\frac{k\pi}{4}\\ x=\frac{2\pi}{3}-k\pi\end{array}\right.$$

5/ $2\sin 4x+3\cos 2x+16\sin^3 x\cos x-5=0$

$$\Leftrightarrow 2\sin 4x+3\cos 2x+8\sin 2x.\sin^2 x-5=0$$

$$\Leftrightarrow 2\sin 4x+3\cos 2x+8\sin 2x.\left(\frac{1-\cos 2x}{2}\right)-5=0$$

$$\Leftrightarrow 2\sin 4x+3\cos 2x+4\sin 2x-2\sin 4x-5=0$$

$$\Leftrightarrow 3\cos 2x+4\sin 2x=5 \Leftrightarrow \frac{3}{5}\cos 2x+\frac{4}{5}\sin 2x=1$$

$$\Leftrightarrow \mathrm{Cos}(2x-\alpha)=1 \Rightarrow x=\frac{\alpha}{2}+k\pi;(k\in ¢);\begin{cases}\cos\alpha=\frac{3}{5}\\ \sin\alpha=\frac{4}{5}\end{cases}$$

- **BTVN NGÀY 06-05**

1/ $Sinx - 4\sin^3 x + \cos x = 0(1)$

$\Leftrightarrow N\hat{e}'u : \cos x = 0 \Rightarrow Sinx - 4\sin^3 x = \pm 3 \neq 0$

$(1) \Leftrightarrow \tan x(1+\tan^2 x) - 4\tan^3 x + 1 + \tan^2 x = 0$

$$\Leftrightarrow \begin{cases} t = \tan x \\ -3t^3 + t^2 + t + 1 = 0 \end{cases} \Leftrightarrow \begin{cases} t = \tan x \\ (t-1)(3t^2 + 2t + 1) = 0 \end{cases} \Leftrightarrow \tan x = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi$$

2/ $\tan x \sin^2 x - 2\sin^2 x = 3(\cos 2x + \sin x \cos x)$

Chia VT, VP cho $\cos^2 x$ *ta có*:

$$\tan^3 x - 2\tan^2 x = 3\frac{(\cos^2 x - \sin^2 x + \sin x \cos x)}{\cos^2 x}$$

$$\Leftrightarrow \tan^3 x - 2\tan^2 x = 3(1 - \tan^2 x + \tan x) \Leftrightarrow \begin{cases} \tan x = t \\ t^3 + t^2 - 3t - 3 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \tan x = t \\ (t+1)(t^2 - 3) = 0 \end{cases} \Leftrightarrow \left[\begin{array}{l} \tan x = -1 \\ \tan x = \pm\sqrt{3} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = -\frac{\pi}{4} + k\pi \\ x = \pm\frac{\pi}{3} + k\pi \end{array}\right.$$

3/ $Sin2x + 2\tan x = 3$

Chia VT, VP cho $\cos^2 x$ *ta có*:

$$2\tan x + 2\tan x(\tan^2 x + 1) = 3(\tan^2 x + 1) \Leftrightarrow \begin{cases} t = \tan x \\ 2t^3 - 3t^2 + 4t - 3 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} t = \tan x \\ (t-1)(2t^2 - t + 3) = 0 \end{cases} \Leftrightarrow \tan x = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi$$

$4/\ Cos^2x-\sqrt{3}\sin 2x=1+\sin^2 x$

Chia VT, VP cho $\cos^2 x$ *ta có:*

$$1-2\sqrt{3}\tan x=2\tan^2 x+1\Leftrightarrow\begin{cases}t=\tan x\\ 2t^2+2\sqrt{3}t=0\end{cases}$$

$$\Leftrightarrow\left[\begin{array}{l}\tan x=0\\ \tan x=-\sqrt{3}\end{array}\right.\Leftrightarrow x=\left[\begin{array}{l}k\pi\\ -\dfrac{\pi}{3}+k\pi\end{array}\right.$$

$5/\ 3\cos^4 x-4\sin^2 x\cos^2 x+\sin^4 x=0$

Chia VT, VP cho $\cos^4 x$ *ta có:*

$$3-4\tan^2 x+\tan^4 x=0\Leftrightarrow\begin{cases}t=\tan x\\ t^4-4t^2+3=0\end{cases}$$

$$\Leftrightarrow\left[\begin{array}{l}\tan^2 x=1\\ \tan^2 x=3\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\pm\dfrac{\pi}{4}+k\pi\\ x=\pm\dfrac{\pi}{3}+k\pi\end{array}\right.$$

- **BTVN NGÀY 07-05**

$1/\ Sinx-\cos x+7\sin 2x=1$

$Coi: t=\sin x-\cos x;(|t|\le\sqrt{2})$

$$\Rightarrow t+7(1-t^2)=1\Leftrightarrow 7t^2-t-6=0\Leftrightarrow\left[\begin{array}{l}\sin x-\cos x=1\\ \sin x-\cos x=\dfrac{6}{7}\end{array}\right.$$

$$\Leftrightarrow\left[\begin{array}{l}\sin\left(x-\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{2}}\\ \sin\left(x-\dfrac{\pi}{4}\right)=-\dfrac{3\sqrt{2}}{7}\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\dfrac{\pi}{2}+k2\pi\\ x=\pi+k2\pi\\ x=\alpha+\dfrac{\pi}{4}+k2\pi\\ x=\dfrac{\pi}{4}-\alpha+k2\pi\end{array}\right.;\sin\alpha=-\dfrac{3\sqrt{2}}{7}$$

$2/\ Sin2x+\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=1$

$Coi: t=\sin x-\cos x;(|t|\le\sqrt{2})$

$$\Rightarrow 1-t^2+t=1\Leftrightarrow\left[\begin{array}{l}t=0\\t=1\end{array}\right.\Leftrightarrow\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=\left[\begin{array}{l}0\\1\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\frac{\pi}{4}+k2\pi\\x=\frac{\pi}{2}+k2\pi\\x=\pi+k2\pi\end{array}\right.$$

$3/\ \text{Tìm}\ m\ cho\ PT: Sin2x+4(\cos x-\sin x)=m\ \ có\ ng_0$

$Coi: t=\cos x-\sin x;(|t|\le\sqrt{2})\Rightarrow 1-t^2+4t=m$

$\Leftrightarrow m=f(t)=-t^2+4t+1\Rightarrow f'(t)=-2t+4>0;\forall|t|\le\sqrt{2}$

$\Rightarrow f(-\sqrt{2})\le m\le f(\sqrt{2})\Leftrightarrow -4\sqrt{2}-1\le m\le 4\sqrt{2}-1$

$4/\ Cos2x+5=2(2-\cos x)(\sin x-\cos x)$

$Cos2x+5=4(\sin x-\cos x)-\sin 2x+\cos 2x+1$

$\Leftrightarrow 4((\sin x-\cos x)-\sin 2x-4=0$

$Coi: t=\sin x-\cos x;(|t|\le\sqrt{2})\Rightarrow 4t-(t^2-1)-4=0\Leftrightarrow t^2-4t+3=0$

$$\Leftrightarrow\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=1\Leftrightarrow\sin\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\Leftrightarrow x=\left[\begin{array}{l}\frac{\pi}{2}+k2\pi\\\pi+k2\pi\end{array}\right.$$

$5/\ Sin^3x+\cos^3x=2(\sin^5x+\cos^5x)$

$\Leftrightarrow Sin^3x\left(1-2\sin^2x\right)+\cos^3x\left(2\cos^2x-1\right)=0$

$\Leftrightarrow \cos 2x\left(\sin x-\cos x\right)\left(\sin^2x-\sin x\cos x+\cos^2x\right)=0$

$\Leftrightarrow \cos 2x=0\Leftrightarrow x=\frac{\pi}{4}+\frac{k\pi}{2}$

- **BTVN NGÀY 08-05**

$$1/\ \ 2\cos 2x - 8\cos x + 7 = \frac{1}{\cos x} (1)$$

$$DK: x \neq \frac{\pi}{2} + k\pi$$

$$(1) \Leftrightarrow \begin{cases} t = \cos x (t \neq) \\ 4t^3 - 8t^2 + 5t - 1 = 0 \end{cases} \Leftrightarrow \left[\begin{array}{l} \cos x = 1 \Rightarrow x = k2\pi \\ \cos x = \frac{1}{2} \Rightarrow x = \pm\frac{\pi}{3} + k2\pi \end{array} \right. ; k \in ¢$$

$$2/\ \ 4\cos^2 x + 3\tan^2 x - 4\sqrt{3}\cos x + 2\sqrt{3}\tan x + 4 = 0 (2)$$

$$DK: x \neq \frac{\pi}{2} + k\pi$$

$$(2) \Leftrightarrow \left(2\cos x - \sqrt{3}\right)^2 + \left(\sqrt{3}\tan x + 1\right)^2 = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \pm\frac{\pi}{6} + k2\pi \\ \tan x = -\frac{1}{\sqrt{3}} \Rightarrow x = -\frac{\pi}{6} + k\pi \end{array} \right. \Leftrightarrow x = -\frac{\pi}{6} + k2\pi \left(k \in ¢\right)$$

$$3/\ \ \sqrt{3 - \cos x} - \sqrt{\cos x + 1} = 2$$

$$\Leftrightarrow \sqrt{3 - \cos x} = \sqrt{\cos x + 1} + 2 \Leftrightarrow 4\sqrt{\cos x + 1} = -2(\cos x + 1)$$

$$Do: \begin{cases} -2(\cos x + 1) \leq 0; \forall x \\ 4\sqrt{\cos x + 1}; \forall x \end{cases} \Rightarrow \cos x + 1 = 0 \Leftrightarrow \cos x = -1 \Leftrightarrow x = \pi + k2\pi \left(k \in ¢\right)$$

$$4/\ \ S\text{in}^3 x - \cos^3 x = \cos 2x.\tan\left(x+\frac{\pi}{4}\right).\tan\left(x-\frac{\pi}{4}\right)$$

$$(\text{sinx-}\cos x)(1+\sin x\cos x) = -\cos 2x \Leftrightarrow (\text{sinx-}\cos x)(1+\sin x\cos x+\text{sinx}+\cos x)=0$$

$$\Leftrightarrow \left[\begin{array}{l} \text{sinx-}\cos x = 0 \Rightarrow \sin\left(x-\frac{\pi}{4}\right)=0 \Leftrightarrow x=\frac{\pi}{4}+k\pi \\ 1+\sin x\cos x+\text{sinx}+\cos x = 0 \Leftrightarrow \begin{cases} t=\text{sinx}+\cos x(|t|\le\sqrt{2}) \\ t+\dfrac{t^2-1}{2}+1=0 \Leftrightarrow t^2+2t+1=0 \Leftrightarrow t=-1 \end{cases} \end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{4}+k\pi \\ \sin\left(x+\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{4}+k\pi \\ x=-\frac{\pi}{2}+k2\pi;(k\in ¢) \\ x=\pi+k2\pi \end{array}\right.$$

$$5/\ \ Cos^2\left(x+\frac{\pi}{3}\right)+Cos^2\left(x+\frac{2\pi}{3}\right)=\frac{1}{2}(\text{sinx}+1)$$

$$\Leftrightarrow \frac{1}{4}\left(\cos x-\sqrt{3}\,\text{sinx}\right)^2+\frac{1}{4}\left(\cos x+\sqrt{3}\,\text{sinx}\right)^2=\frac{1}{2}(\text{sinx}+1)$$

$$\Leftrightarrow \frac{1}{2}\left(1+2\sin^2 x\right)=\frac{1}{2}(\text{sinx}+1) \Leftrightarrow 2\sin^2 x-\sin x=0 \Leftrightarrow \left[\begin{array}{l} \text{sinx}=0 \\ \text{sinx}=\frac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k2\pi \\ x=\frac{\pi}{6}+k2\pi\ \ ;k\in ¢ \\ x=\frac{5\pi}{6}+k2\pi \end{array}\right.$$

- **BTVN NGÀY 10-05**

Bài 1:

Tìm các nghiệm thuộc khoảng (2π/5; 6π/7) của phương trình:

$$\sqrt{3}\sin 7x - \cos 7x = \sqrt{2}$$

Giải:

$$PT \Leftrightarrow \frac{\sqrt{3}}{2}\sin 7x - \frac{1}{2}\cos 7x = \frac{\sqrt{2}}{2} \Leftrightarrow \sin\left(7x - \frac{\pi}{6}\right) = \sin\frac{\pi}{4} \Leftrightarrow \left[\begin{array}{l} x = \frac{5\pi}{84} + \frac{k2\pi}{7} \\ x = \frac{11\pi}{84} + \frac{k2\pi}{7} \end{array}\right. ;(k \in ¢)$$

$$*Khi: x = \frac{5\pi}{84} + \frac{k2\pi}{7} \Rightarrow \frac{2\pi}{5} < \frac{5\pi}{84} + \frac{k2\pi}{7} < \frac{6\pi}{7} \Leftrightarrow \frac{2}{5} - \frac{5}{84} < \frac{2k}{7} < \frac{6}{7} - \frac{5}{84}$$

$$\Leftrightarrow k = 2 \Leftrightarrow x_1 = \frac{53\pi}{84}$$

$$*Khi: x = \frac{11\pi}{84} + \frac{k2\pi}{7} \Rightarrow \frac{2\pi}{5} < \frac{11\pi}{84} + \frac{k2\pi}{7} < \frac{6\pi}{7} \Leftrightarrow \frac{2}{5} - \frac{11}{84} < \frac{2k}{7} < \frac{6}{7} - \frac{11}{84}$$

$$\Leftrightarrow k = 1,2 \Leftrightarrow x_2 = \frac{35\pi}{84}; x_3 = \frac{59\pi}{84}$$

Bài 2:

Tìm các nghiệm thuộc khoảng (π/2; 3π) của phương trình:

$$\sin\left(2x + \frac{5\pi}{2}\right) - 3\cos\left(x - \frac{7\pi}{2}\right) = 1 + 2\sin x$$

Giải:

$$PT \Leftrightarrow Sin\left(2x + 2\pi + \frac{\pi}{2}\right) - 3\cos\left(x + \frac{\pi}{2} - 4\pi\right) = 1 + 2\sin x$$

$$\Leftrightarrow \cos 2x + 3\sin x = 1 + 2\sin x \Leftrightarrow 1 - 2\sin^2 x = 1 - \sin x$$

$$\Leftrightarrow 2\sin^2 x - \sin x = 0 \Leftrightarrow \left[\begin{array}{l} \sin x = 0 \Rightarrow x = k\pi \\ \sin x = \dfrac{1}{2} \Rightarrow \left[\begin{array}{l} x = \dfrac{\pi}{6} + k2\pi \\ x = \dfrac{5\pi}{6} + k2\pi \end{array}\right. \end{array}\right.$$

$$\Leftrightarrow Do\ x \in (\frac{\pi}{2}; 3\pi) \Rightarrow x_1 = \pi; x_2 = 2\pi; x_3 = \frac{13\pi}{6}; x_4 = \frac{5\pi}{6}; x_5 = \frac{17\pi}{6}$$

Bài 3:

Tìm m để phương trình sau có 4 nghiệm thuộc khoảng (-π;7π/3):

$$\sin x + m\cos x = m$$

Giải:

$$PT \Leftrightarrow \sin x = m(1-\cos x) \Leftrightarrow \left[\begin{array}{l} \cos x = 1 \\ m = \dfrac{\sin x}{1-\cos x} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = 0 \text{ và } x = 2\pi \\ m = \dfrac{\sin x}{1-\cos x} (*) \end{array}\right.$$

Vậy để phương trình ban đầu có 4 nghiệm thì (*) phải có 2 nghiệm phân biệt thuộc khoảng (-π;7π/3).

Nhưng số nghiệm của (*)thuộc khoảng (-π;7π/3) lại chính là số giao điểm của đường thẳng y=m với đồ thị (C) có phương trình:

$$y = \frac{\sin x}{1-\cos x} \text{ trên } D = \left(-\pi; \frac{7\pi}{3}\right)$$

$$\text{Xét hàm}: y' = \frac{\cos x - 1}{(1-\cos x)^2} < 0\ \forall x \in D$$

Dựa vào bảng biến thiên ta có: $m \geq \sqrt{3}; m \leq 0\ \ PT\ \text{có}\ 4ng_0$

……………………**Hết**…………………

BT Viên môn Toán hocmai.vn

Trịnh Hào Quang